SAMPLE QUESTION PAPER (STANDARD) - 01

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

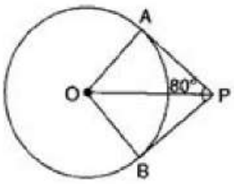
1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If the line segment joining the points (3, -4) and (1, 2) is trisected at points P (a, -2) and $Q\left(\frac{5}{3}, b\right)$. Then, **[1]**

a) $a = \frac{8}{3}, b = \frac{2}{3}$
b) $a = \frac{7}{3}, b = 0$
c) $a = \frac{2}{3}, b = \frac{1}{3}$
d) $a = \frac{1}{3}, b = 1$

2. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then find ∠POA **[1]**



a) 40°
b) 100°
c) 50°
d) 60°

3. An unbiased die is thrown once. The probability of getting a number between 2 and 6 is **[1]**

a) $\frac{1}{2}$
b) $\frac{2}{5}$
c) $\frac{1}{3}$
d) $\frac{2}{3}$

4. If P is a point on x-axis such that its distance from the origin is 3 units, then the coordinates of a point Q on OY such that OP = OQ, are **[1]**

a) (0, 0)
b) (0, -3)

c) (0, 3) d) (3, 0)

5. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have [1]

a) a unique solution
b) infinitely many solutions
c) no solution
d) exactly two solutions

6. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by [1]

a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
b) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$
c) $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$
d) $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

7. If P(E) denotes the probability of an event E then [1]

a) $0 \leq P(E) \leq 1$
b) $-1 \leq P(E) \leq 1$
c) P(E) < 0
d) P(E) > 0

8. The maximum volume of a cone that can be carved out of a solid hemisphere of radius 'r' is [1]

a) πr^3
b) $\frac{2}{3}\pi r^3$
c) $\frac{1}{3}\pi r^3$
d) $\frac{1}{3}\pi r^2 h$

9. Two dice are thrown together. The probability of getting the same number on both dice is [1]

a) $\frac{1}{6}$
b) $\frac{1}{12}$
c) $\frac{1}{3}$
d) $\frac{1}{2}$

10. If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then q = [1]

a) 8
b) -16
c) 16
d) -8

11. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? [1]

a) -2 or 0
b) 0 only
c) 2 or 0
d) 2 or -2

12. If $\tan\theta = \frac{a}{b}$, then $\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta}$ is [1]

a) $\frac{a+b}{a-b}$
b) $\frac{a^2-b^2}{a^2+b^2}$
c) $\frac{a-b}{a+b}$
d) $\frac{a^2+b^2}{a^2-b^2}$

13. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a = [1]

a) 1
b) 2
c) 4
d) 3

14. If P($\frac{a}{3}$, 4) is the mid-point of the line segment joining the points Q (-6, 5) and R (-2, 3), then the value of a is [1]

a) 12
b) -12
c) -4
d) -6

15. From the top of a building 60m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. The height of the tower is [1]

a) 40 m
b) 60 m

c) 45 m

d) 50 m

16. For the following distribution: **[1]**

Class	60-70	70-80	80-90	90-100	100-110
Frequency	13	10	15	8	11

the lower limit of the modal class is

a) 80

b) 100

c) 90

d) 70

17. The LCM of $2^3 \times 3^2$ and $2^2 \times 3^3$ **[1]**

a) 2×3^2

b) $2^3 \times 3^3$

c) $2^2 \times 3^2$

d) $2^2 \times 3$

18. The area of the triangle formed by the lines x = 3, y = 4 and x = y is **[1]**

a) 1 sq. unit

b) 1/2 sq. unit

c) None of these

d) 2sq. unit

19. **Assertion (A):** 3 is a rational number. **[1]**

Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** In the △ABC, AB = 24 cm, BC = 10 cm and AC = 26 cm, then △ABC is a right-angle triangle. **[1]**

Reason (R): If in two triangles, their corresponding angles are equal, then the triangles are similar.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A bag contains 2 green, 3 red and 4 black balls. A ball is taken out from the bag at random. Find the probability that the selected ball is: **[2]**

a. not green

b. not black

22. Is the pair of linear equations consistent? Justify your answer. **[2]**

–3x– 4y = 12, 4y + 3x = 12

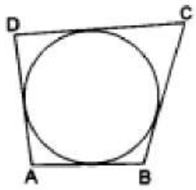
23. Find the zeroes of the quadratic polynomial $x^2 + 5x + 6$ and verify the relationship between the zeroes and the coefficients. **[2]**

24. If the distance between the points (3, 0) and (0, y) is 5 units and y is positive, then what is the value of y? **[2]**

OR

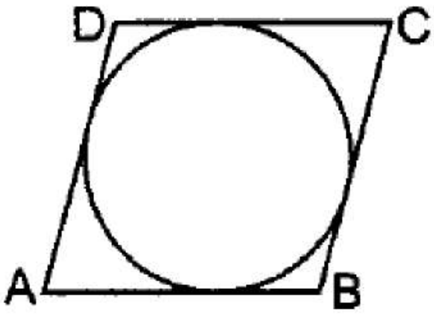
Find the lengths of the medians AD and BE of ΔABC whose vertices are A(7, -3), B(5, 3) and C(3, -1).

25. In the adjoining figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD. **[2]**



OR

Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above prove the following: A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



Section C

26. If tan A = n tanB and sin A = m sinB, then prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$ **[3]**

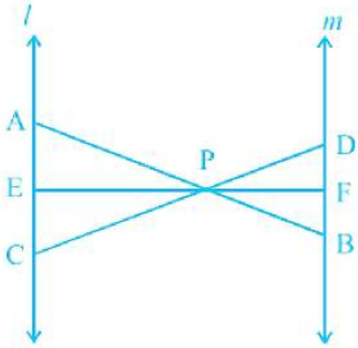
27. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number. **[3]**

28. If (x–k) is the HCF of $\left(2x^2 - kx - 9\right)$ and $x^2 + x - 12$, find the value of k. **[3]**

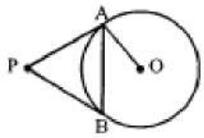
OR

Find the HCF and LCM of the following positive integers by applying the prime factorization method: 15, 55, 99

29. In the figure, l || m and line segments AB, CD, and EF are concurrent at point P. Prove that $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$. **[3]**

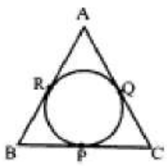


30. In the adjoining figure PA and PB are tangents to the circle with centre O. If $\angle APB = 60^\circ$, then find $\angle OAB$. **[3]**



OR

ABC is an isosceles triangle in which AB = AC, circumscribed about a circle, as shown in the adjoining figure. Prove that the base is bisected at the point of contact.



31. If a tower 30m high, casts a shadow $10\sqrt{3}m$ long on the ground, then what is the angle of elevation of the sun? **[3]**

Section D

32. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of a larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[5]**

OR

Solve for y:

$\frac{y+3}{y-2} - \frac{1-y}{y} = \frac{17}{4}; y \neq 0 \cdot 2$

33. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then other two sides are divided in the same ratio. By using this theorem, prove that in $\triangle$ ABC if DE $\parallel$ BC then $\frac{AD}{BD} = \frac{AE}{AC}$. **[5]**

34. A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14) **[5]**

i. minor sector

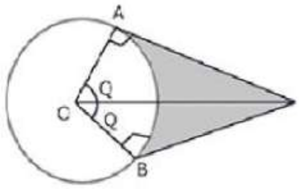
ii. major sector

iii. minor segment

iv. major segment

OR

An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area.



35. The following table gives the distribution of the life time of 400 neon lamps: **[5]**

Lite time (in hours)	**Number of lamps**
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life time of a lamp.

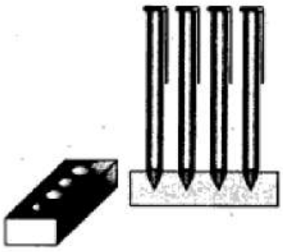
Section E

36. **Read the text carefully and answer the questions:** **[4]**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4

cm.



(i) The volume of the cuboidal part.

(ii) The volume of wood in the entire stand.

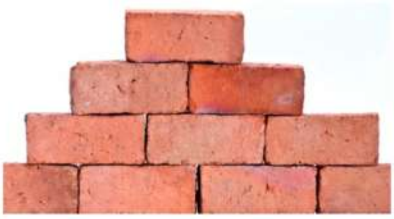
(iii) Total volume of conical depression.

OR

If the cost of wood used is ₹10 per cm^3, then the total cost of making the pen stand.

37. **Read the text carefully and answer the questions:** **[4]**

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such as way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

(i) In how many rows, 360 bricks are placed?

(ii) How many bricks are there in the top row?

OR

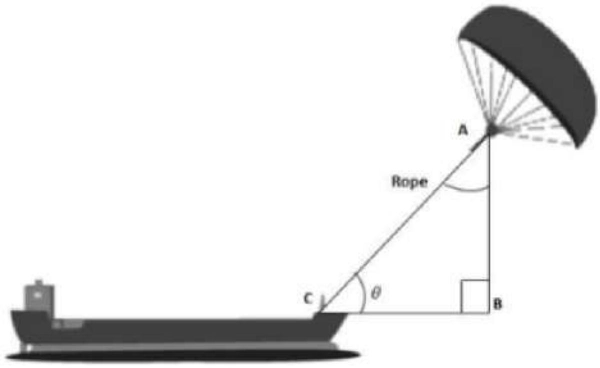
If which row 26 bricks are there?

(iii) How many bricks are there in 10^{th} row?

38. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if sin θ = cos(θ - 30°), where θ and θ - 30° are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^{o})$, where θ and $3\theta - 30^{o}$ are acute angles, then find the value of θ.

Solution

SAMPLE QUESTION PAPER (STANDARD) - 01

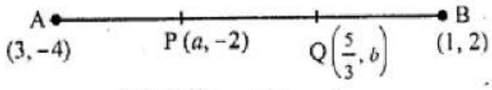
Class 10 - Mathematics

Section A

1. **(b)** $a=\frac{7}{3}, b=0$

   Explanation: The points P(a, -2) and Q $\left(\frac{5}{3}, b\right)$ trisect the line segment joining the points A(3, -4) and B(1, 2)

   $\because$ P divides AB in the ratio 1 : 2

   Then, $a=\frac{1\times1+2\times3}{1+2}=\frac{1+6}{3}=\frac{7}{3}$

   and Q divides AB in the ratio 2 : 1, then

   $b=\frac{2\times2+1\times(-4)}{2+1}=\frac{4-4}{3}=0$

   $\therefore a=\frac{7}{3}, b=0$

2. **(c)** 50°

   Explanation: Here $\angle$OAP = 90°

   And $\angle\mathrm{OPA}=\frac{1}{2}\angle\mathrm{BPA}$ [Centre lies on the bisector of the angle between the two tangents]

   $\Rightarrow\angle\mathrm{OPA}=\frac{1}{2}\times80^{\circ}=40^{\circ}$

   Now, in triangle OPA,

   $\angle\mathrm{OAP}+\angle\mathrm{OPA}+\angle\mathrm{POA}=180^{\circ}$

   $\Rightarrow$90° + 40° + $\angle$POA = 180°

   $\Rightarrow$ $\angle$POA = 50°

3. **(a)** $\frac{1}{2}$

   Explanation: Number of numbers between 2 and 6 on a dice = {3, 4, 5}, = 3

   Number of possible outcomes = 3

   Number of Total outcomes = 6

   $\therefore$ Required Probability = $\frac{3}{6}=\frac{1}{2}$

4. **(c)** (0, 3)

   Explanation: P is a point on x-axis and its distance from 0 is 3

   Co-ordinates of P will be (3, 0)

   Q is a point on OY such that OP = OQ

   Co-ordinates of Q will be (0, 3)

5. **(c)** no solution

   Explanation: Given, equations are

   x + 2y + 5 = 0, and

   - 3x - 6y + 1 = 0.

   Comparing the equations with general form:

   $a_1x + b_1y + c_1 = 0$

   $a_2x + b_2y + c_2 = 0$

   Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$

   And $a_2 = -3$, $b_2 = -6$, $c_2 = 1$

   Taking the ratio of coefficients to compare

   $\frac{a_1}{a_2}=\frac{-1}{3}, \frac{b_1}{b_2}=\frac{-1}{3}, \frac{c_1}{c_2}=\frac{5}{1}$

   So $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

   This represents a pair of parallel lines.

   Hence, the pair of equations has no solution.

6. **(a)** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

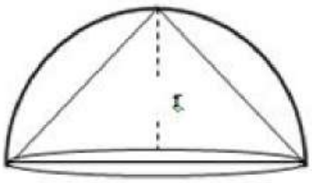
Explanation: we know that the midpoint formula = $\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

7. **(a)** $0 \leq P(E) \leq 1$

Explanation: The probability of any event is always positive. It could be at the least equal to zero but not less than that. The probability of sure event at the maximum could be equal to 1 so probability lies between 0 and 1 both included.

8. **(c)** $\frac{1}{3}\pi r^3$

Explanation: 

Volume of cone = $\frac{1}{3}\pi r^2 h$

Here height of the carved out cone = Radius of the hemisphere

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3$

9. **(a)** $\frac{1}{6}$

Explanation: Here 2 dice are thrown together.

$\therefore$ Number of total outcomes = $6 \times 6 = 36$

Number which should come together are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) = 6 pairs

Therefore, probablity = $\frac{1}{6}$

10. **(c)** 16

Explanation: $x^2 + bx + 12 = 0$

$\because$ 2 is its root, then 2 will satisfy it

$\therefore (2)^2 + b \times 2 + 12 \Rightarrow 4 + 2b + 12 = 0$

$\Rightarrow 2b + 16 = 0 \Rightarrow b = \frac{-16}{2} = -8$

Now equation

$x^2 + bx + q = 0$, has equal rooots then

$D = 0 \Rightarrow b^2 - 4q = 0$

$\Rightarrow (-8)^2 - 4q = 0 \Rightarrow 64 = 4q$

$\Rightarrow q = 16$

11. **(d)** 2 or -2

Explanation: Since the roots are equal, we have D = 0.

$\therefore 36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

12. **(d)** $\frac{a^2+b^2}{a^2-b^2}$

Explanation: $\tan\theta = \frac{a}{b}$

$\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta}+b\frac{\cos\theta}{\cos\theta}}{a\frac{\sin\theta}{\cos\theta}-b\frac{\cos\theta}{\cos\theta}}$ (Dividing by $\cos\theta$)

$= \frac{a\tan\theta+b}{a\tan\theta-b} = \frac{a\times\frac{a}{b}+b}{a\times\frac{a}{b}-b}$

$= \frac{\frac{a^2}{b}+b}{\frac{a^2}{b}-b} = \frac{\frac{a^2+b^2}{b}}{\frac{a^2-b^2}{b}}$

$= \frac{a^2+b^2}{b} \times \frac{b}{a^2-b^2}$

$= \frac{a^2+b^2}{a^2-b^2}$

13. **(c)** 4

Explanation: LCM (a, 18) = 36

HCF (a, 18) = 2

We know that the product of numbers is equal to the product of their HCF and LCM.

Therefore,

18a = 2(36)

$a = \frac{2(36)}{18}$

a = 4

14. **(b)** -12

Explanation:

Given, P is the mid - point of the line segment joining the points Q and R

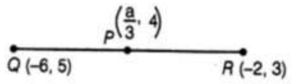
Where;

P = $\left(\frac{a}{3}, 4\right)$

Q = (- 6, 5)

R = (- 2, 3)

Shown in the figure given below;



∴ Mid - point of QR = P$\left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$ = (-4, 4)

P = (- 4, 4)

Since, midpoint of line segment having points (x_1, y_1) and (x_2, y_2);

= $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$

But given coordinates of mid - point P is $\left(\frac{a}{3}, 4\right)$;

∴ $\left(\frac{a}{3}, 4\right)$ = (- 4, 4)

On comparing the coordinates, we get

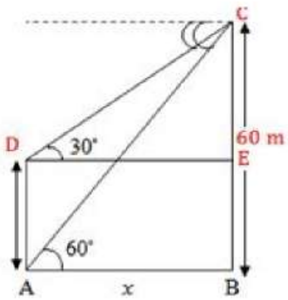
$\frac{a}{3}$ = - 4

∴ a = - 12

Hence, the required value of a = - 12

15. **(a)** 40 m

Explanation:



In triangle CDE,

$\tan 30^\circ = \frac{60-h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\Rightarrow x = \sqrt{3}(60-h)$ meters.....(i)

Again, in triangle CAB,

$\tan 60^\circ = \frac{60}{x}$

$\Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = \frac{60}{\sqrt{3}}$ meters .....(ii)

From eq. (i), and (ii), we get,

$\sqrt{3}(60-h) = \frac{60}{\sqrt{3}}$

$\Rightarrow 60 - h = 20$

$\Rightarrow h$ = 40 meters

16. **(a)** 80

Explanation: In the given data, Maximum frequency is 15.

Therefore, the modal class is 80 - 90.

The lower limit of the modal class is 80.

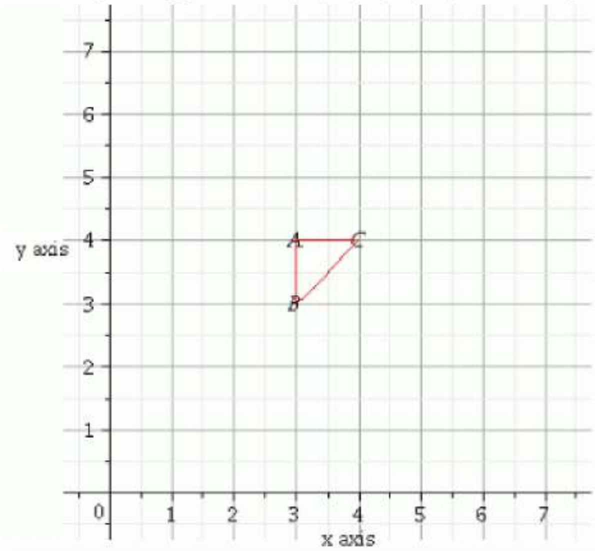
17. **(b)** $2^3 \times 3^3$

Explanation: L.C.M. of $2^3 \times 3^2$ and $2^2 \times 3^3$ is the product of all prime numbers with the greatest power of every given number, hence it will be $2^3 \times 3^3$

18. **(b)** 1/2 sq. unit

Explanation: Given x = 3, y = 4 and x = y

We have plotting points as (3,4), (3,3), (4,4) when x = y



Therefore, area of $\triangle ABC = \frac{1}{2}$(Base $\times$ Height) = $\frac{1}{2}$ (AB $\times$ AC) = $\frac{1}{2}$ $(1 \times 1) = \frac{1}{2}$

Area of triangle ABC is $\frac{1}{2}$ square units.

19. **(c)** A is true but R is false.

Explanation: Here, reason is not true.

$\sqrt{9} = \pm 3$, which is not an irrational number.

A is true but R is false.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: We have,

$AB^2 + BC^2 = (24)^2 + (10)^2$

$= 576 + 100 = 676 = AC^2$

$AB^2 + BC^2 = AC^2$

ABC is a right-angled triangle.

Also, two triangles are similar if their corresponding angles are equal. So, both A and R are true but R is not the correct explanation of A.

Section B

21. Total number of balls in the bag = $2 + 3 + 4 = 9$

i. No of balls which are not green = 3 + 4 = 7

Probability(not green) = $\frac{7}{9}$

ii. No of balls which are not black = 3 + 2 = 5

Probability(not black) $= \frac{5}{9}$

22. Conditions for pair of linear equations to be consistent is

$a_1/a_2 \neq b_1/b_2$.. [unique solution]

and $a_1/a_2 = b_1/b_2 = c_1/c_2$ ...[coincident or infinitely many solutions]

Comparing the given pair of linear equations

- 3x - 4y - 12 = 0 and 4y + 3x - 12 = 0

with standard form we get:

$a_1 = -3$, $b_1 = -4$, $c_1 = -12$;

And $a_2 = 3$, $b_2 = 4$, $c_2 = -12$;

$a_1/a_2 = -3/3 = -1$

$b_1/b_2 = -4/4 = -1$

$c_1/c_2 = -12/-12 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Hence, the pair of linear equations has no solution, i.e., inconsistent.

23. Here $f(x) = x^2 + 5x + 6$

$= x^2 + 3x + 2x + 6$

$= x(x+3) + 2(x+3)$

$= (x+3+(x+2)$

$f(x) = 0$ if $x+3=0$ or $x+2=0$

So the zeros of f(x) are -3 and -2

for $= x^2 + 5x + 6$

a=1, b=5, c=6

Sum of zeros=-3-2=-5=-$\frac{5}{1} = -\frac{b}{a}$

Product of zeros $= (-3)(-2) = 6 = \frac{6}{1} = \frac{c}{a}$

Hence the relationship between coefficients and zeros is verified

24. Distance between (3, 0) and (0, y) is 5 units

$\therefore \sqrt{(0-3)^2 + (y-0)^2} = 5$

$\sqrt{9 + y^2} = 5$

$9 + y^2 = 25 \Rightarrow y^2 = 25 - 9 = 16 = (\pm 4)^2$

$\therefore y = \pm 4$

But y is positive

$\therefore y = 4$

OR

According to the question, A(7, -3), B(5, 3) and C(3, -1).

AD and BE are medians of ΔABC.

D is the mid-point of BC and

E is the mid-point of AC

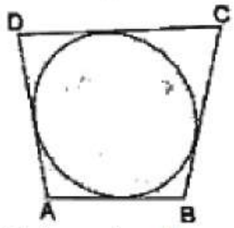
$\therefore$ Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right) = (4, 1)$

Coordinates of $E = \left(\frac{7+3}{2}, \frac{-3-1}{2}\right) = \left(\frac{10}{2}, \frac{-4}{2}\right) = (5, -2)$

Now, median $AD = \sqrt{(4-7)^2 + (1+3)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5\ units$

And, median $BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0 + (-5)^2} = \sqrt{25} = 5\ units$

25.



We know that when a quadrilateral circumscribes a circle then the sum of opposite sides is equal to the sum of other opposite sides.

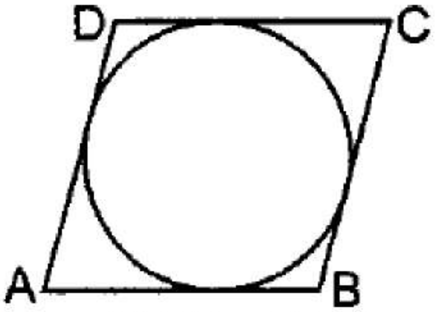
AB + CD = AD + BC

$\Rightarrow 6 + 8 = AD + 9$

$\Rightarrow AD = 5$ cm.

OR

Let ABCD be the quadrilateral circumscribing the circle with centre O. The quadrilateral touches the circle at points P, Q, R, S.



To prove: $AB + CD = AD + BC$

proof: lengths of tangents drawn from an external point are equal

Hence, $AP = AS$ ...(i)

$BP = BQ$ ...(ii)

$CR = CQ$ ...(iii)

$DR = DS$ ...(iv)

Adding (i) + (ii) + (iii) + (iv), we get

$AB + BP + CR + DR = AS + BQ + CQ + DS$

$AB + CD = AD + BC$

Hence proved

Section C

26. Given,

tan A = n tan B

$\Rightarrow$ tanB = $\frac{1}{n}$tan A

$\Rightarrow$ cotB = $\frac{n}{\tan A}$ ....(1)

Also given,

sin A = m sin B

$\Rightarrow$ sin B = $\frac{1}{m}$sin A

$\Rightarrow$ cosec B = $\frac{m}{\sin A}$ .....(2)

We know that, cosec^2B - $\cot^2$B = 1, hence from (1) & (2) :-

$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \quad \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$

$\Rightarrow$ $m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow$ $m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow$ $m^2 - 1 = n^2\cos^2 A - \cos^2 A$

$\Rightarrow$ $m^2 - 1 = (n^2 - 1)\cos^2 A$

$\Rightarrow \quad \frac{m^2-1}{n^2-1} = \cos^2 A$

27. Let the one's digit be 'a' and ten's digit be 'b'.

Given, two digit number is 4 times the sum of its digits and twice the product of the digits.

$\Rightarrow$ 10b + a = 4(a + b)

$\Rightarrow$ a = 2b

Also, 10b + a = 2ab

Substituting value of a.

$\Rightarrow$ 10b + 2b = 2 $\times$ 2b $\times$ b

$\Rightarrow$ b = 3

Thus, a = 6

Thus, the number is 36.

28. Let p(x) $= 2x^2 - kx - 9$ and $q(x) = x^2 + x - 12$

Since (x-k) is the HCF of both p(x) and q(x),

therefore (x-k) divides both p(x) and q(x) exactly.

$\Rightarrow x-k$ is a factor of both $p(x)$ and $q(x)$

$\therefore$ By factor theorem p(k) = 0

and also q(k) = 0

Now p(k) = 0

$\Rightarrow 2k^2 - kk - 9 = 0 \Rightarrow k^2 - 9 = 0$ Using Identity $a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow (k-3)(k+3) = 0 \Rightarrow k = 3, -3$ ....(i)

Again q(k) = 0

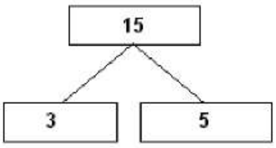
$\Rightarrow k^2 + k - 12 = 0 \Rightarrow k^2 + 4k - 3k - 12 = 0$

$\Rightarrow k(k+4) - 3(k+4) \Rightarrow (k+4)(k-3) = 0$

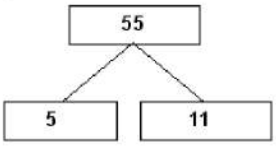
$\Rightarrow k = -4, 3$ ....(ii)

Hence from (i) and (ii), k = 3

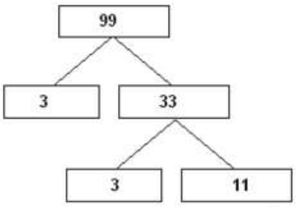
OR



so, 15 = 3 × 5



So, 55 = 5 × 11



So, 99 = $3 \times 3 \times 11 = 3^2 \times 11$

Therefore,

HCF (15 ,55, 99) = 1

LCM (15, 55, 99) = $3^2 \times 5 \times 11 = 495$.

29. Given,

l || m and line segment AB, CD and EF are concurrent at point P.

To prove $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

In $\triangle$APC and BPD,

$\angle$APC = $\angle$BPD [vertically opposite angles]

$\angle$PAC = $\angle$PBD [alternate angles]

$\therefore \triangle APS \sim \triangle BPD$ [by AAA similarity criterion]

Then,

$\frac{AP}{PB} = \frac{AC}{BD} = \frac{PC}{PD}$ ....(i)

In $\triangle$APE and $\triangle$BPF,

$\angle$APE = $\angle$BPF [vertically opposite angles]

$\angle$PAE = $\angle$PBF [alternate angles]

$\therefore \triangle APE \sim \triangle BPF$ [by AAA similarity criterion]

Then,

$\frac{AP}{PB} = \frac{AE}{BF} = \frac{PE}{PF}$ ....(ii) [by basic propertionality theorem]

In $\triangle$PEC and $\triangle$PED,

$\angle$EPC = $\angle$FPD [Vertically opposite angles]

$\angle$PCE = $\angle$PDF [alternate angles]

$\therefore \triangle PEC \sim \triangle PFD$ [by AAA similarity criterion]

Then,

$\frac{PE}{PF} = \frac{PC}{PD} = \frac{EC}{FD}$ ....(iii) [by basic proportionality theorem]

From Eqs. (i), (ii) and (iii), we get

$\frac{AP}{PB} = \frac{AC}{BD} = \frac{AE}{BF} = \frac{PE}{PF} = \frac{EC}{FD}$

$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

Hence proved.

30.

<u>Construction:</u> Join OB

We know that the radius and tangent are perpendicular at the point of contact.

$\because \angle OBP = \angle OAP = 90^\circ$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360^\circ$

$\Rightarrow 240^\circ + \angle AOB = 360^\circ$

$\Rightarrow \angle AOB = 120^\circ$

Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\Rightarrow 120^\circ + 2\angle OAB = 180^\circ$

$\Rightarrow \angle OAB = 30^\circ$

OR

The lengths of tangents drawn from an external point to a circle are equal.

$\therefore AR = AQ$ ...(i)

$BR = BP$ ...(ii)

$CQ = CP$...(iii)

Given that ABC is an isosceles triangle AB = AC.

Subtract AP on both sides, we obtain

$AB - AR = AC - AR$

$\Rightarrow AB - AR = AC - AQ$ (from (i)

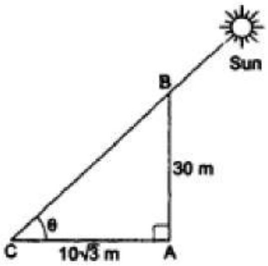
$\therefore BR = CQ.$

$\Rightarrow BP = CQ$ (from (ii)

$\Rightarrow BP = CP$ (from (iii))

$\therefore$ BC is bisected at the point of contact R.

31. Let AB be the pole and let AC be its shadow.



Let the angle of elevation of the sun be θ°.

Then, $\angle ACB = \theta, \angle CAB = 90^\circ$.

$AB = 30m$ and AC = $10\sqrt{3}m.$

From right ΔCAB, we have

$\tan\theta = \frac{AB}{AC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\Rightarrow \theta = 60^\circ$

Section D

32. Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = (x - 10) hr

Part of the tank filled by a smaller pipe in 1 hour $= \frac{1}{x}$

Part of the tank filled by the larger pipe in 1 hour $= \frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. So $\frac{75}{8}$ hours, multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e 1.

$\frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$

$\Rightarrow 75(2x-10) = 8x^2 - 80x$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

Now for factorizing the above quadratic equation, two numbers are to be found such that their product is equal to 750×8 and their sum is equal to 230

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-25)(8x-30) = 0$

$\Rightarrow x = 25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours.

As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 - 10 = 15 hours respectively.

OR

Given equation, $\frac{y+3}{y-2} - \frac{1-y}{y} = \frac{17}{4}$

$\Rightarrow \quad \frac{y(y+3)-(1-y)(y-2)}{y(y-2)} = \frac{17}{4}$

$\Rightarrow \frac{(y^2+3y)-(-y^2+3y-2)}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \frac{y^2+3y+y^2-3y+2}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \quad \frac{2y^2+2}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \quad 4(2y^2+2) = 17(y^2-2y)$

$\Rightarrow \quad 8y^2 + 8 = 17y^2 - 34y$

$\Rightarrow \quad 9y^2 - 34y - 8 = 0$

$\Rightarrow \quad 9y^2 - 36y + 2y - 8 = 0$

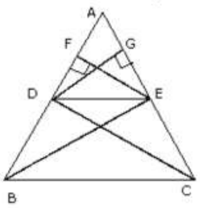
$\Rightarrow \quad 9y(y-4) + 2(y-4) = 0$

$\Rightarrow \quad (y-4)(9y+2) = 0$

$\Rightarrow \quad y - 4 = 0 \quad or \quad 9y + 2 = 0$

$\Rightarrow \quad y = 4 \quad or \quad y = -\frac{2}{9}$

$\therefore \quad y = 4, -\frac{2}{9}$

33. 

Given: In $\triangle$ ABC if DE $\parallel$ BC intersect AB at D and AC at E.

To Prove: $\frac{AD}{BD} = \frac{AE}{AC}$

Construction: Draw EF $\perp$ AB and DG $\perp$ AC and join DC and BE.

Proof: $ar\triangle ADE = \frac{1}{2}AD \times EF$

$ar\triangle DBE = \frac{1}{2}DB \times EF$

$\therefore \frac{ar\triangle ADE}{ar\triangle DBE} = \frac{\frac{1}{2}AD \times EF}{\frac{1}{2}DB \times EF} = \frac{AD}{DB}$ ....(i)

Similarly, $\therefore \frac{ar\triangle ADE}{ar\triangle DEC} = \frac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \frac{AE}{EC}$

Since $\triangle$DBE and $\triangle$DEC are on the same base and between the same parallels

$\therefore$ ar($\triangle$DBE) = ar ($\triangle$DEC)

$\Rightarrow \frac{1}{ar(\triangle DBE)} = \frac{1}{ar(\triangle DEC)}$

$\therefore \frac{ar\triangle ADE}{ar\triangle DBF} = \frac{ar\triangle ADE}{ar\triangle DFC}$

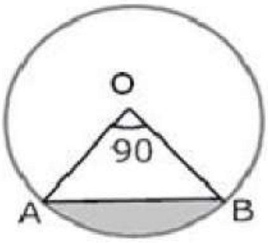
$\Rightarrow \frac{AD}{DB} = \frac{AB}{EC}$

$\because$ DE || BC

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC} \left[\because \frac{p}{q} = \frac{r}{s} \Rightarrow \frac{p}{p+q} = \frac{r}{r+s}\right]$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

34. 

i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector

= $\pi(10)^2$ - $\frac{90}{360}$ $\pi(10)^2$ = 3.14 (100) - $\frac{1}{4}$(3.14) (100)

= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle$OAB $= \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\,(\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

= $\frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

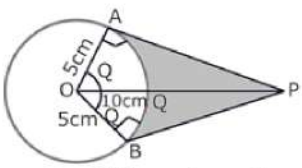
iv. Area of major segment = Area of the circle - Area of minor segment

= $\pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

OR



$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta$ = 60^o

$\Rightarrow \angle$AOB = 2 $\times$ θ = 120^o

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times5}{360}cm = \frac{10\pi}{3}cm \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360}cm^2 = \frac{25\pi}{3}cm^2 \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$[\because AP = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}\ cm]$

Hence, shaded area $= 25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

35.

Life time	**Number of lamps (f_i)**	**Cumulative frequency**
1500-2000	14	14
2000-2500	56	14 + 56 = 70
2500-300	60	70 + 60 = 130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290
4000-4500	62	290 + 62 = 352
4500-5000	48	352 + 48 = 400
	400	

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

Here,

l = Lower limit of median class

F = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

Lower limit (l) of median class = 3000

Frequency (f) of median class 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median = $3000 + \left(\frac{200-130}{86}\right) \times 500$

$= 3000 + \frac{70 \times 500}{86}$

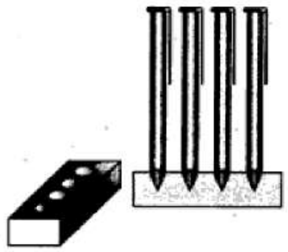
= 3406.98

Section E

36. **Read the text carefully and answer the questions:**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525\text{cm}^3$

(ii) Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

$\therefore$ Volume of four conical depressions

$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3 = 1.47\text{cm}^3$

(iii)$\because$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depressions

$= 525 - 1.47 = 523.53\text{cm}^3$

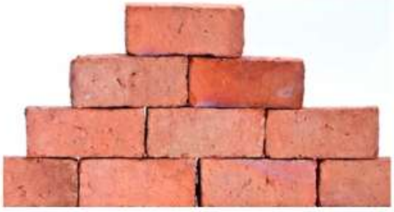
OR

Cost of wood per cm^3 = ₹10

Total cost of making pen stand = 10 × 523.53 = ₹5235.3

37. **Read the text carefully and answer the questions:**

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such as way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

(i) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360 \ \{S_n = \frac{n}{2}(2a + (n-1)d)\}$

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow 720 = 60n - n^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorization]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

$\Rightarrow (n-16) = 0$ or $(n-45) = 0$

$\Rightarrow n = 16$ or $n = 45$

Hence, number of rows is either 45 or 16.

n = 45 not possible so n = 16

$a_{45} = 30 + (45-1)(-1) \ \{a_n = a + (n-1)d\}$

$= 30 - 44 = -14$ [$\because$ The number of logs cannot be negative]

Hence the number of rows is 16.

(ii) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

Number of bricks on top row are $n = 16$,

$a_{16} = 30 + (16 - 1)(-1) \ \{a_n = a + (n - 1)d\}$

= 30 - 15 = 15

Hence, and number of bricks in the top row is 15.

OR

Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

$a_n = 26$, a = 30, d = -1

$a_n = a + (n - 1)d$

$\Rightarrow 26 = 30 + (n - 1) \times -1$

$\Rightarrow 26 - 30 = -n + 1$

$\Rightarrow n = 5$

Hence 26 bricks are in 5th row.

(iii)Number of bricks in the bottom row = 30. in the next row = 29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.

Suppose number of rows is n, then sum of number of bricks in n rows should be 360

Number of bricks in 10th row a = 30, d = -1, n = 10

$a_n = a + (n - 1)d$

$\Rightarrow a_{10} = 30 + 9 \times -1$

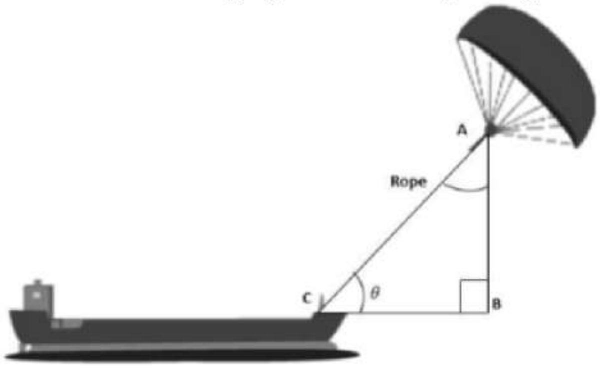
$\Rightarrow a_{10} = 30 - 9 = 21$

Therefore, number of bricks in 10th row are 21.

38. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin \theta = \cos(\theta - 30^o)$

$\cos (90^o - \theta) = \cos (\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^o} = \frac{200}{\frac{\sqrt{3}}{2}} = \frac{200 \times 2}{\sqrt{3}}$ = 230.94 m

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o} = \frac{150}{\frac{1}{2}}$ = 150 × 2 = 300 m

(iii)$\sin \theta = \cos(3\theta - 30^o)$

$\cos(90^o - \theta) = \cos(3\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$